# Announcements

- Final exam is 1 week from today, Wednesday 17th of December
- No notes, books, calculators, neighbors
- ~90-100 questions, including some bonus questions
- 2 hours, 10-12 am
- This room!

# Astronomy 103

Review for Final Exam

Quick Course Overview
and Practice Problems

# Outline I

- Scientific notation and powers of ten
- Units, scale of the Universe
- The celestial sphere: motions of the stars, the seasons, phases of the Moon, eclipses
- Copernican revolution: Kepler's laws, Newton's law of gravity
- Light and matter: the electromagnetic spectrum, emission and absorption lines, thermal radiation
- Telescopes: different types for different wavelengths
- The Sun: structure, power source

# Outline II

- Stars: luminosity, apparent brightness, temperature
- The Hertzprung-Russell diagram
- Stellar masses with binary stars
- The interstellar medium: gas, dust and star formation
- Stellar evolution: evolutionary stages of stars of different masses, white dwarfs, supernovae
- Neutron stars and black holes
- The solar system: terrestrial and Jovian planets, comets and asteroids, formation of the solar system
- Extrasolar planets
- The Milky Way galaxy: structure, mass (dark matter), the Galactic Center

# Outline III

- Normal and Active Galaxies
  - The Hubble Sequence: spiral, elliptical and irregular galaxies
  - The distance ladder
  - The distribution of galaxies in space: clusters of galaxies
  - Hubble's law
  - Active Galactic Nuclei: quasars, radio galaxies, accretion onto supermassive black holes

# Outline IV

- Galaxies and dark matter
  - Evidence for dark matter: galaxy rotation curves, masses of galaxy clusters, gravitational lensing
  - Galaxy collisions, galaxy evolution
- Cosmology
  - The universe on the largest scales
  - The density, geometry and fate of the universe
  - The Big Bang and the Early Universe
    - Big Bang nucleosynthesis
  - The Cosmic Microwave Background
  - Inflation
  - The formation of large scale structure

# Things Involving Math

- Kepler's third law
- Newton's law of gravity
- Temperature and peak wavelength of thermal (blackbody) radiation
- Conversion of mass to energy in stars, stellar luminosities and lifetimes
- Brightness and distance
- Hubble's law

**We will do examples of these today**

# Kepler's Third Law:

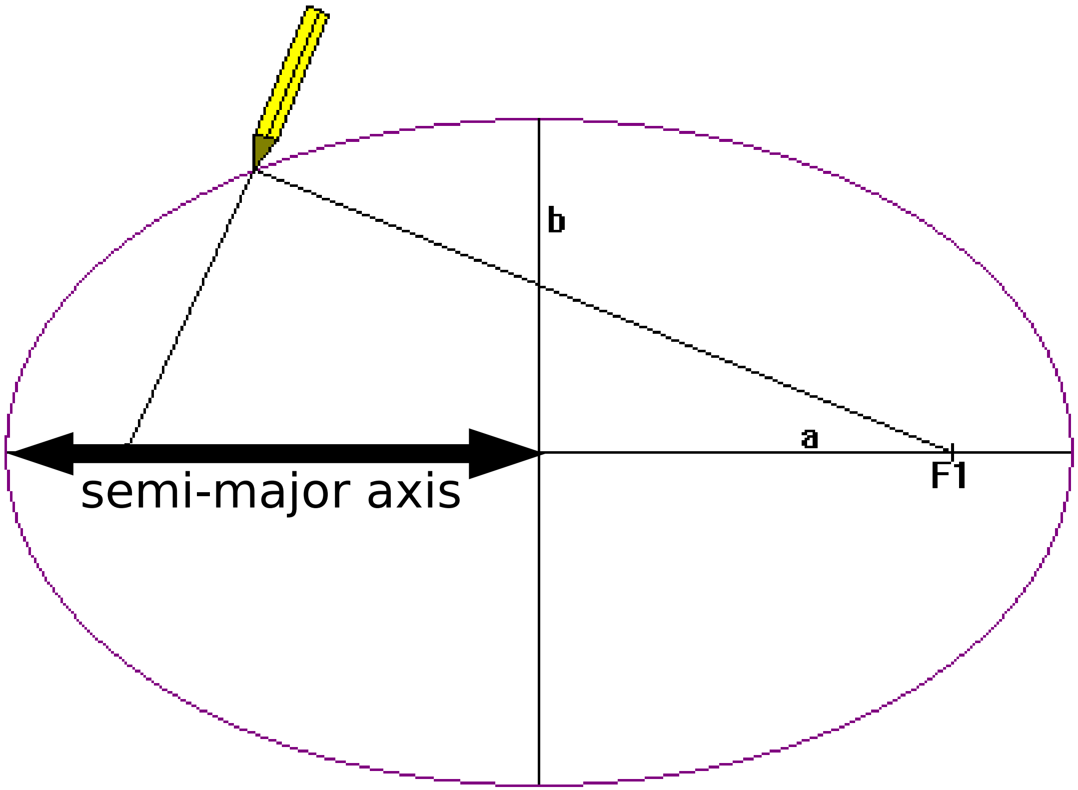
The period of each planet is related to its average distance from the Sun by the formula

$$a^3 = P^2$$

a = planet's average distance from Sun (semimajor axis of ellipse) in AU

P = planet's period in years

# Reminder: semi-major axis



# Using Kepler's Third Law: Examples

$$a^3 = P^2$$

- Earth has a period of 1 year and a semi-major axis of 1 AU:
  $1^3 = 1^2 = 1$.

- A planet is at 2 A.U. What is its period?
  Solve Kepler's 3rd law for P: $P = \sqrt{a^3} = a^{3/2}$

→ The period of the planet is $P = \sqrt{2^3} = 2.8$ years

# Using Kepler's Third Law: Examples

$$a^3 = P^2$$

- Earth has a period of 1 year and a semi-major axis of 1 AU:
  $1^3 = 1^2 = 1$.

- A planet is at 0.1 A.U. What is its period?
  Solve Kepler's 3rd law for P: $$P = \sqrt{a^3} = a^{3/2}$$

→ The period of the planet is $P = \sqrt{0.1^3} = 0.03$ years

# Using Kepler's Third Law: Examples

- Compute period from semi-major axis:
- a=1 AU → p = 1 year
- a=2 AU → p = 2.8 years
- a=3 AU → p = 5.2 years
- a=0.1 AU → p = 0.03 years

Notice that planets with a>1 AU have periods greater than 1 year, and planets at a<1 AU have periods less than 1 year.

# Using Kepler's Third Law: Examples

$$a^3 = P^2$$

- Earth has a period of 1 year and a semi-major axis of 1 AU: $1^3 = 1^2 = 1$.

- A comet has a period of 1000 years. What is its distance from the Sun?
  Solve Kepler's 3rd law for a: $$a = \sqrt[3]{P^2} = P^{2/3}$$

  → The distance of the comet from the sun is $$a = \sqrt[3]{1000^2} = 100^{\mathrm{AU}}$$

# **Newton's law of gravity**:

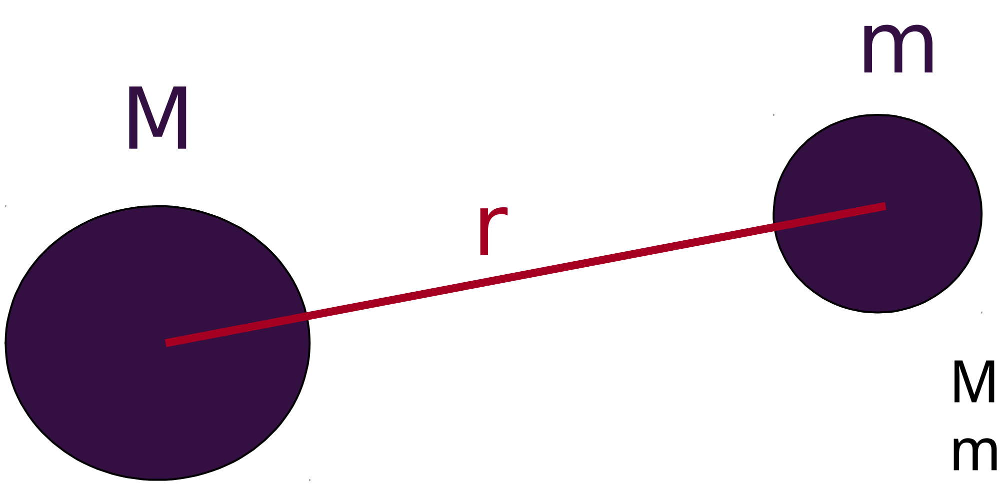
Any two objects in the universe attract each other with a force given by

$$F = G\frac{M\,m}{r^2}$$

With
$r$ in meters
$m$ in kg

$F$ in Newtons, where
1 Newton = 4.5 lb

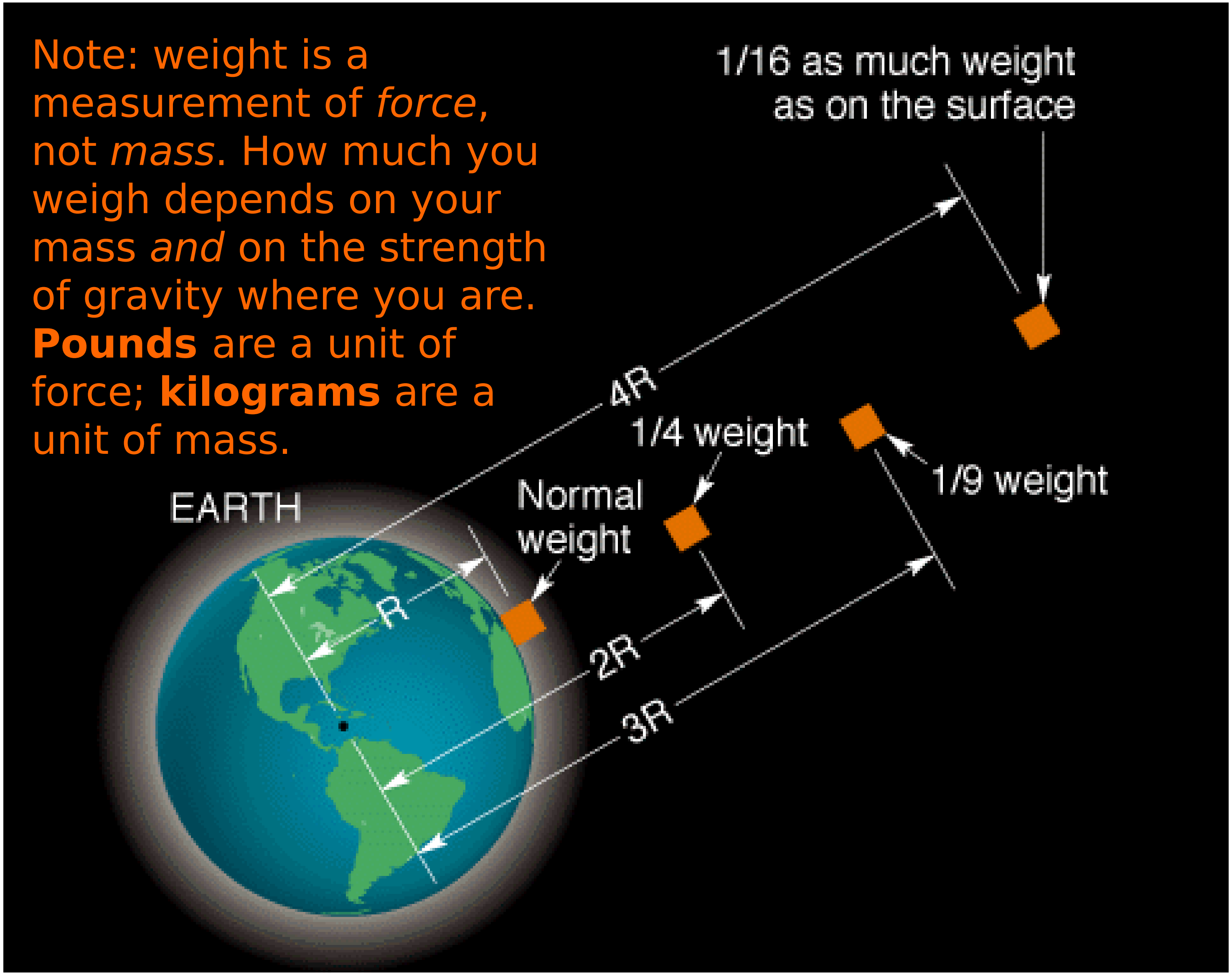
$G$: Newton's gravitational constant
$G = 7\text{x}10^{-11}$




M: mass of object 1
m: mass of object 2
r: distance between the objects

# Force of gravity on an object proportional to $1/r^2$

Note: weight is a measurement of *force*, not *mass*. How much you weigh depends on your mass *and* on the strength of gravity where you are. **Pounds** are a unit of force; **kilograms** are a unit of mass.



# Using Newton's law of Gravity

$$F = G\frac{Mm}{r^2}$$

An astronaut weighs 140 pounds on Earth. How much would she weigh on a planet that is the same size as Earth but ½ the mass?

A 35 pounds

C 140 pounds

B 70 pounds

D 280 pounds

# Using Newton's law of Gravity

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An astronaut weighs 140 pounds on Earth. How much would she weigh on a planet that is the same size as Earth but ½ the mass?

A 35 pounds

B 70 pounds

C 140 pounds

D 280 pounds

$$F = G\frac{Mm}{r^2}$$

Mass of new planet is ½ mass of Earth:

$$F_{new} = G\frac{(0.5\mathrm{M})m}{r^2} = 0.5G\frac{Mm}{r^2} = 0.5F_{old}$$

So the gravitational force on the new planet is half as strong as on Earth, and the astronaut weighs half as much.

Remember that weight is gravitational force! The **mass** of the astronaut *m* does not change!

# Using Newton's law of Gravity

$$F = G\frac{Mm}{r^2}$$

An astronaut weighs 150 pounds on Earth. How much would she weigh on a planet that is the same size as Earth but 0.1 times the mass?

A 1.5 pounds

B 15 pounds

C 150 pounds

D 1500 pounds

# Using Newton's law of Gravity

$$F = G\frac{Mm}{r^2}$$

An astronaut weighs 150 pounds on Earth. How much would she weigh on a planet that is the same size as Earth but 0.1 times the mass?

A 1.5 pounds

C 150 pounds

B 15 pounds

D 1500 pounds

# Using Newton's law of Gravity

$$F = G\frac{Mm}{r^2}$$

An astronaut weighs 120 pounds on Earth. How much would she weigh on a planet that is the same mass as Earth but double the radius?

A 30 pounds

C 240 pounds

B 60 pounds

D 480 pounds

# Using Newton's law of Gravity

$$F = G\frac{Mm}{r^2}$$

An astronaut weighs 120 pounds on Earth. How much would she weigh on a planet that is the same mass as Earth but double the radius?

A 30 pounds

C 240 pounds

B 60 pounds

D 480 pounds

$$F = G\frac{M\,m}{r^2}$$

Radius of new planet is twice the radius of Earth:

$$F_{new} = G\frac{M\,m}{(2\mathrm{r})^2} = \frac{1}{2^2}G\frac{Mm}{r^2} = \frac{1}{4}F$$

The gravitational force on the new planet is ¼ as strong as on Earth, and the astronaut weighs ¼ as much.

# Using Newton's law of Gravity

$$F = G\frac{Mm}{r^2}$$

An astronaut weighs 100 pounds on Earth. How much would she weigh on a planet that is the same mass as Earth but 1/3 the radius?

A 11 pounds

C 300 pounds

B 33 pounds

D 900 pounds

# Using Newton's law of Gravity

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An astronaut weighs 100 pounds on Earth. How much would she weigh on a planet that is the same mass as Earth but 1/3 the radius?

A 11 pounds

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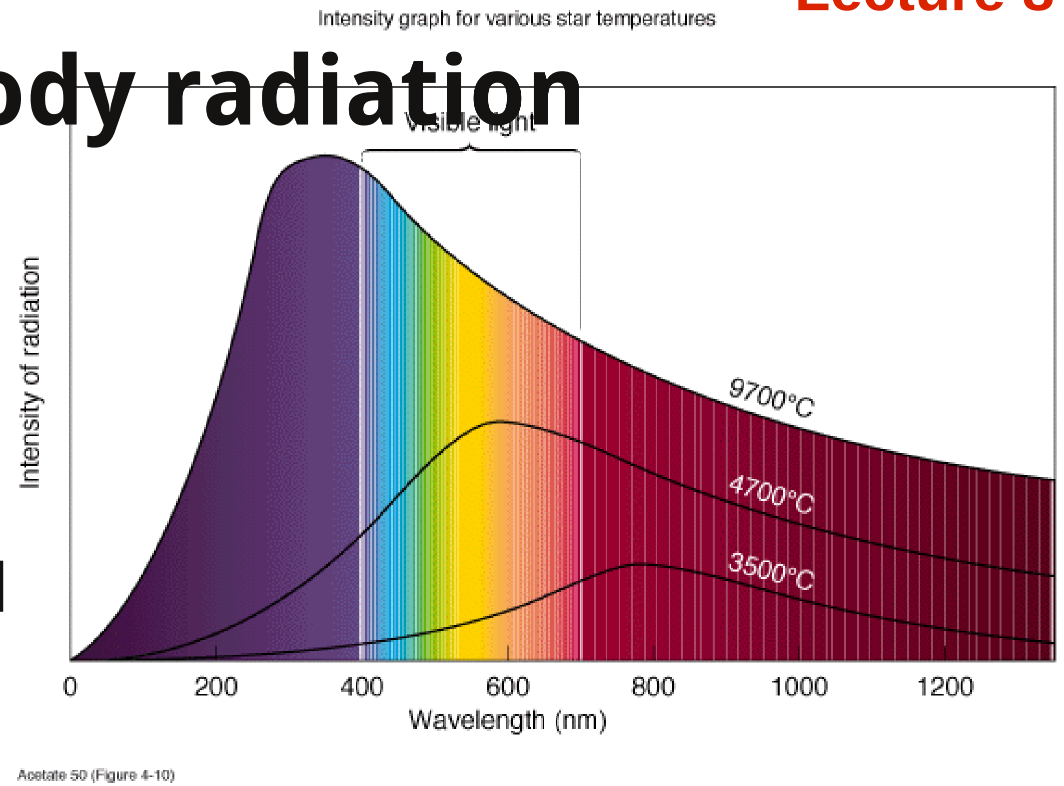
# Blackbody radiation




The radiation that something emits because of its temperature is called **thermal or blackbody radiation**, and we can use the peak wavelength of the blackbody curve to determine the temperature of an object.

Nothing radiates as a perfect blackbody, but stars come close!

So **hotter stars emit more radiation, and the radiation peaks at a shorter wavelength.**

With wavelength λ measured in nanometers and **T in degrees Kelvin**, the relation between the average wavelength of light emitted by a hot object and the temperature of the object is

$$\lambda = \frac{3\times10^6}{T}\ \mathrm{nm}$$

To 2 decimal-place accuracy and with λ measured in cm

$$\lambda = \frac{0.29}{T}\ \mathrm{cm}$$

Example: Milwaukee at 170 C = 290 K
Peak wavelength is 10,000 nm: infrared

$$\lambda = \frac{3\times10^6}{T}\ \text{nm}$$

A star has a temperature of 2000 K. What is the peak wavelength of light that it emits?

$$\lambda = \frac{3\times10^6}{2000}\ \text{nm} = 1500\ \text{nm}$$

Recall that the peak wavelength of light from the Sun is at about 480 nm. Is a star with peak wavelength 1500 nm hotter or colder than the Sun?

A Hotter

B Colder

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A Hotter

B Colder

Longer peak wavelength = colder temperature

$$\lambda = \frac{3 \times 10^6}{T}\ \mathrm{nm}$$

A star emits at a peak wavelength of 300 nm. What is the temperature of its surface?

A 0.9 K

B 9000 K

C 10,000 K

D 9 x 108 K

$$\lambda = \frac{3\times10^{6}}{T}\ \mathrm{nm}$$

A star emits at a peak wavelength of 300 nm. What is the temperature of its surface?

A 0.9 K

B 9000 K

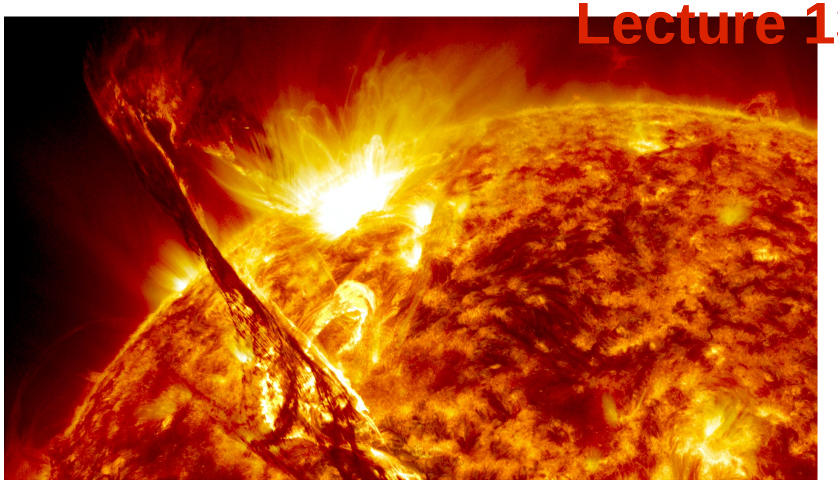
C 10,000 K

D 9 x 108 K

$$T = \frac{3\times10^{6}}{\lambda}\ \mathrm{K}$$

If wavelength in nm

# Stars



Energy source:
nuclear fusion of
hydrogen into helium

When 4 hydrogen atoms are fused into one helium atom, 0.7% of their mass is turned into energy:

$$E=mc^2$$

# Stars

The luminosity of the Sun is $4 \times 10^{26}$ watts.

So the sun produces $4 \times 10^{26}$ watt-seconds of energy every second.

How much mass does it fuse into energy to do this?

**$E=mc^2$, so $m=E/c^2$**

The speed of light is $c = 3 \times 10^8$ m/s

$m = 4 \times 10^{26} / (3 \times 10^8)2 = 4.4 \times 10^9$ kg

→ The sun turns $4.4 \times 10^9$ kg of mass into energy every second.

# Stars

The Sun turns $4.4 \times 10^9$ kg of mass into energy every second.

How much hydrogen does the Sun fuse into helium in one second to do this?

When 4 hydrogen atoms are fused into one helium atom, 0.7% of their mass is turned into energy.

**mass turned into energy = total mass x 0.007**

So the total amount of hydrogen fused is the amount of mass turned into energy divided by 0.007, which is the amount of mass turned into energy x 143.

The Sun fuses $143 \times (4.4 \times 10^9$ kg$) = 6.3 \times 10^{11}$ kg of hydrogen into helium in one second.

# Stars

The Sun fuses 143 x (4.4 x $10^9$ kg) = 6.3 x $10^{11}$ kg of hydrogen into helium in one second.

**How long will it live?**

The mass of the Sun is 2 x $10^{30}$ kg, and the mass of its core which will be fused into helium is about 10% of that. This is the total amount of fuel available.

total amount of fuel = 0.1 x 2 x $10^{30}$ kg = 2 x $10^{29}$ kg

The lifetime is the total amount of fuel divided by the amount of fuel consumed per second.

So the lifetime of the Sun is (2 x $10^{29}$ kg)/(6.3 x $10^{11}$ kg/s) = 3.2 x $10^{17}$ seconds = $10^{10}$ years

# Stars

We can repeat this calculation for other stars, given their masses and luminosities.

$$\text{Lifetime} = \frac{\text{mass of core}}{\text{amount of fuel consumed per sec}}$$

The total mass changed to energy is proportional to the mass of the star. If a star has 5 times the mass of the Sun it will change 5 times as much mass to energy in its lifetime.

The mass changed to energy each second is proportional to the luminosity of the star. If a star has 1000 times the luminosity of the Sun, it will change 1000 times as much mass to energy each second.

A star with 5 times the mass and 1000 times the luminosity of the Sun then has a lifetime 5/1000 = 0.005 times as long.

# Lifetimes of Stars

$$\text{lifetime of star} = 10^{10}\,\text{years}\,\frac{M}{L}$$

The total mass changed to energy is proportional to the mass of the star. If a star has 5 times the mass of the Sun it will change 5 times as much mass to energy in its lifetime.

The mass changed to energy each second is proportional to the luminosity of the star. If a star has 1000 times the luminosity of the Sun, it will change 1000 times as much mass to energy each second.

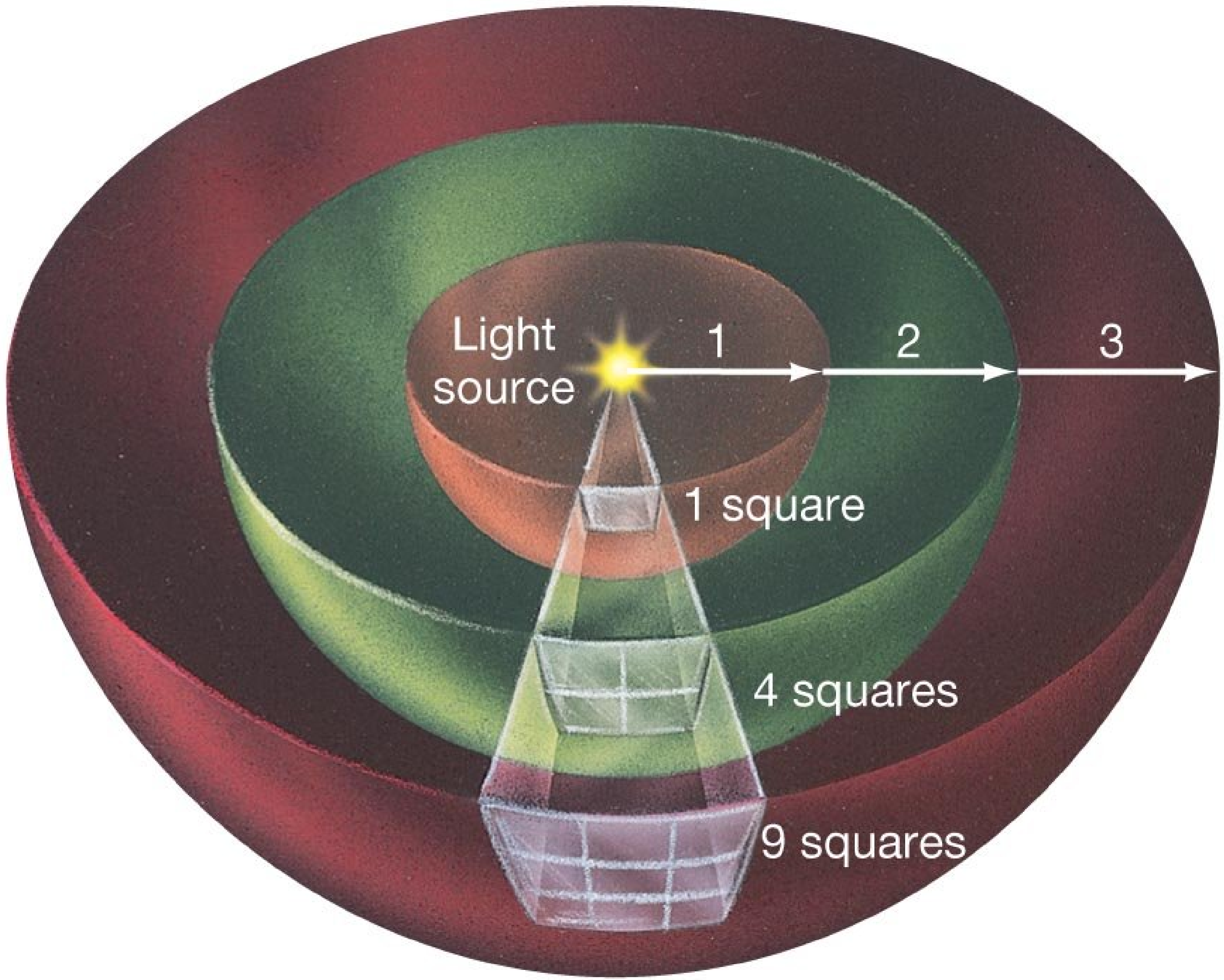
A star with 5 times the mass and 1000 times the luminosity of the Sun then has a lifetime 5/1000 = 0.005 times as long.

A 5 solar mass stars lives for $10^{10}$ x 5/1000 = 5 x $10^{7}$ years.

# Brightness and distance

As light moves away from a star (or light bulb) its energy is spread out over a larger area. That area is proportional to $d^2$.

So if you move twice as far away from a star, it will look 4 times fainter.



$$B \text{ proportional to } 1/\text{d}^2$$

*or*

$$B_2 = B_1 \times \frac{d_1^2}{d_2^2}$$

- Example: The brightness of sunlight at the Earth is 1400 watts/meter$^2$. What is the brightness of sunlight at Saturn, 10 AU from the Sun?

- Example: The brightness of sunlight at the Earth is 1400 watts/meter2. What is the brightness of sunlight at Saturn, 10 AU from the Sun?

$$B \text{ proportional to } 1/d^2$$

- Saturn is 10 times farther away from the Sun than the Earth, so sunlight is $1/10^2 = 1/100$ times brighter.
- The brightness of sunlight on Saturn is $1400/100 = 14$ watts/meter$^2$. This is why the outer planets are cold!

# A few more examples

1) What is the brightness of the sun at 40 A.U. if it is 1400 watts/m$^2$ at 1 A.U?

$$B_2 = B_1 \times \frac{d_1^2}{d_2^2}$$

$$= 1400 \times \frac{1^2}{40^2} = 0.875 \text{ watts/m}^2$$

# A few more examples

1) What is the brightness of the sun at 40 A.U. if it is 1400 watt/m2 at 1 A.U?

2) How about at 100 A.U.?

$$B_2 = B_1 \times \frac{d_1^2}{d_2^2}$$

$$= 1400 \times \frac{1^2}{100^2} = 0.14 \text{ watts/m}^2$$

Another example: If the Sun has an apparent brightness of 0.0014 watts/m2, how far away is it?

$$B_2 = B_1 \times \frac{d_1^2}{d_2^2}, \text{ so}$$

$$d_2^2 = d_1^2 \times \frac{B_1}{B_2}$$

$$= (1\ \text{A.U.})^2 \times \frac{1400}{0.0014} = 10^6\ \text{A.U.}^2 \text{ and}$$

$$d_2 = \sqrt{10^6} = 1000\ \text{A.U.}$$

# Hubble's law

Galaxies outside our local group of galaxies are moving away from us, because the universe is expanding! Their speed increases with their distance from us according to the Hubble law:

$$v = H \times d$$

v = velocity in km/s

d = distance in megaparsecs
(1 Mpc = $3.26 \times 10^6$ light years)

The Hubble constant: H = 70 km/s/Mpc

# Hubble's Law

- $v = H \times d$ is called the Hubble Law and H is the Hubble constant.
- To determine it, we need to measure the velocity of many many galaxies, and their distances
- Velocities are easy, from the Doppler shift
- Distances are harder: we use Cepheid variable stars and other methods

# Hubble's Law

**$v = H \times d$, H = 70 km/s/Mpc**

Let's do a few examples.

A galaxy is 100 Mpc away. How fast is it moving away from us?

- $v = H \times d = 70 \times 100 = 7000$ km/s

A galaxy is 2000 Mpc away. How fast is it moving away from us?

- $v = H \times d = 70 \times 2000 = 140{,}000$ km/s

**v = H × d, H = 70 km/s/Mpc**

Let's do a few examples.

- A galaxy is 100 Mpc away. How fast is it moving away from us?
→ v = H × d = 70 × 100 = 7000 km/s

- A galaxy is 2000 Mpc away. How fast is it moving away from us?
→ v = H × d = 70 × 2000 = 140,000 km/s

# Hubble's Law

***v* = *H* × *d*, H = 70 km/s/Mpc**

Let's do a few examples.

A galaxy is moving at 700 km/s away from us. How far away is it?

- $d = v/H$ = 700/70 = 10 Mpc

A galaxy is moving at 21,000 km/s away from us. How far away is it?

- $d = v/H$ = 21,000/70 = 300 Mpc

**v = H × d, H = 70 km/s/Mpc**

Let's do a few examples.

- A galaxy is moving at 700 km/s away from us. How far away is it?
  → d = v/H = 700/70 = 10 Mpc
- A galaxy is moving at 21,000 km/s away from us. How far away is it?
  → d = v/H = 21,000/70 = 300 Mpc